

DIRECT DETECTION RATES OF DARK MATTER COUPLED TO DARK ENERGY

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We investigate the effect of a coupling between dark matter and dark energy on the rates for the direct detection of dark matter. The magnitude of the effect depends on the strength κ of this new interaction relative to gravity. The resulting isothermal velocity distribution for dark matter in galaxy halos is still Maxwell-Boltzmann (M-B), but the characteristic velocity and the escape velocity are increased by $\sqrt{1+\kappa^2}$. We adopt a phenomenological approach and consider values of κ near unity. For such values we find that: (i) The (time averaged) event rate increases for light WIMPs, while it is somewhat reduced for WIMP masses larger than 100 GeV. (ii) The time dependence of the rate arising from the modulation amplitude is decreased compared to the standard M-B velocity distribution. (iii) The average and maximum WIMP energy increase proportionally to $1+\kappa^2$, which, for sufficiently massive WIMPs, allows the possibility of designing experiments measuring γ rays following nuclear de-excitation.

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INTRODUCTION

The combined MAXIMA-1 [1], BOOMERANG [2], DASI [3] and COBE/DMR Cosmic Microwave Background (CMB) [4] observations imply that the Universe is flat [5] and most of its energy content is exotic [6]. These results have been confirmed and improved by the recent WMAP data [7]. The deduced cosmological expansion is consistent with the luminosity distance as a function of redshift of distant supernovae [8]–[10]. According to the scenario favored by the observations there are various contributions to the energy content of our Universe. The most accessible energy component is baryonic matter, which accounts for $\sim 5\%$ of the total energy density. A component that has not been directly observed is cold dark matter (CDM): a pressureless fluid that is responsible for the growth of cosmological perturbations through gravitational instability. Its contribution to the total energy density is estimated at $\sim 25\%$. The dark matter is expected to become more abundant in extensive halos, that stretch up to 100–200 kpc from the center of galaxies. The component with the biggest contribution to the energy density has an equation of state similar to that of a cosmological constant and is characterized as dark energy. The ratio $w = p/\rho$ is negative and close to -1 . This component is responsible for $\sim 70\%$ of the total energy density and induces the observed acceleration of the Universe [8]–[10]. The total energy density of our Universe is believed to take the critical value consistent with spatial flatness.

Since a non-exotic component cannot exceed 40% of the CDM [11], there is room for a component consisting of exotic weakly interacting massive particles (WIMPs). Supersymmetry naturally provides candidates for these dark matter constituents [12, 13]. In the most favored scenario of supersymmetry, the lightest supersymmetric particle (LSP) can be described as a Majorana fermion, a linear combination of the neutral components of the gauginos and higgsinos [12]–[14]. In most calculations the neutralino is assumed to be primarily a gaugino, usually a bino. Even though there exists firm indirect evidence for a halo of dark matter in galaxies from the observed rotational curves, it is essential to detect such matter directly [12]–[15]. Until dark matter is actually detected, we will not be able to exclude the possibility that the rotation curves result from a modification of the laws of nature as we currently view them. The direct detection will also reveal the nature of the constituents of dark matter.

The possibility of direct detection, however, depends on the nature of the dark matter constituents. Since the WIMPs are expected to be very massive ($m_{WIMP} \gtrsim 30$ GeV) and extremely non relativistic with average kinetic energy $\langle T \rangle \simeq 50$ KeV ($m_{WIMP}/100$ GeV), they are not likely to excite the

nucleus. As a result, they can be directly detected mainly via the recoiling of a nucleus (A, Z) in elastic scattering. The event rate for such a process can be computed from the following ingredients:

1. An effective Lagrangian at the elementary particle (quark) level obtained in the framework of the prevailing particle theory. For supersymmetry this is achieved as described in refs. [14, 16], for example.
2. A well defined procedure for transforming the amplitude obtained using the previous effective Lagrangian from the quark to the nucleon level, i.e. a quark model for the nucleon. This step in SUSY models is non-trivial, since the obtained results depend crucially on the content of the nucleon in quarks other than u and d.
3. Knowledge of the relevant nuclear matrix elements [17, 18], obtained with reliable many-body nuclear wave functions. Fortunately, in the case of the scalar coupling, which is viewed as the most important, the situation is a bit simpler, as only the nuclear form factor is needed.
4. Knowledge of the WIMP density in our vicinity and its velocity distribution. Since the essential input here comes from the rotational curves, dark matter candidates other than the LSP (neutralino) are also characterized by similar parameters.

In the past various velocity distributions have been considered for the dark matter gas in our galaxy. The most popular one is the isothermal Maxwell-Boltzmann (M-B) velocity distribution with $\langle v^2 \rangle = 3v_d^2 \simeq 3v_0^2/2$, where $v_d^2 = \langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle$ and v_0 is the velocity of the sun around the galaxy, i.e. $v_0 \simeq 220$ km/s. Extensions of the M-B distribution have also been considered, in particular these that are axially symmetric with enhanced dispersion in the galactocentric direction [19, 20]. In such distributions an upper cutoff $v_{esc} \simeq 2.84 v_0$ is introduced by hand, in order to eliminate velocities above the escape velocity.

Non-isothermal models have also been considered. Among these one should mention the ones with late infall of dark matter into the galaxy, i.e caustic rings [21]–[25], dark matter orbiting the Sun [26], Sagittarius dark matter [27]. The velocity distribution has also been obtained in "adiabatic" models employing the Eddington proposal [28]–[31]. In such an approach, given the density of matter, one can obtain a mass distribution that depends both on the velocity and the gravitational potential. Evaluating this distribution in a given point in space, e.g. in our vicinity, one obtains the velocity distribution at that point in a self-consistent manner. Unfortunately this approach is applicable only if the density of matter is spherically symmetric.

In the present work we will consider another variant of the isothermal M-B distribution, that results when the dark matter interacts with the dark energy [32, 33]. The difficulty with explaining the very small value of the cosmological constant that could induce the present acceleration has motivated the suggestion that this energy component is time dependent [34, 35]. In the simplest realization, it is connected to a scalar field ϕ with a very flat potential. The vacuum energy associated with this field is the dark energy that drives the acceleration. If such a field affects the cosmological evolution today, its effective mass must be of the order of the Hubble scale, or smaller.

It is conceivable that there is a coupling between dark matter and the field responsible for the dark energy [36]. In such a scenario it may be possible to resolve the coincidence problem, i.e. the reason behind the comparable present contributions from the dark matter and the dark energy to the total energy density. The presence of an interaction between dark matter and the scalar field responsible for the dark energy has consequences that are potentially observable. The cosmological implications depend on the form of the coupling, as well as on the potential of the field [37]. If the scale for the field mass is set by the present value of the Hubble parameter, then the field is effectively massless at distances of the order of the galactic scale. Its coupling to the dark matter particles results in a long range force that can affect the details of structure formation [38]–[41].

The attraction between dark matter particles mediated by the scalar field is expected to modify the distribution and velocity of dark matter particles in halos, with implications for dark matter searches. A careful analysis indicates that the distribution remains Maxwell-Boltzmann, but with a potentially larger characteristic velocity [32, 33]. This has two consequences:

- The total detection rate is reduced for large WIMP masses (above 100 GeV). This occurs because the velocity distribution is shifted to higher values. As a result, such a distribution tends to favor a high energy transfer to the nucleus. The nuclear form factor tends to suppress the high energy transfer components, resulting in an overall suppression.
- The modulation effect, i.e. the periodic dependence of the rate on the Earth's motion, is reduced. This is unfortunate, because the modulation is viewed as a good signature against the background.
- As the average WIMP velocity increases, the average WIMP energy increases as well. The kinetic energy becomes

$$\langle T \rangle \approx 50(1 + \kappa^2) \text{ KeV} \frac{m_{WIMP}}{100 \text{ GeV}}. \quad (1)$$

Thus, for $m_{WIMP} = 200 \text{ GeV}$, one finds $\langle T \rangle \simeq 0.32$ and 1.3 MeV for $\kappa^2 = 1$ and 3 respectively. Since a value $\kappa^2 \simeq 1$ cannot be excluded from the available constraints, there is, in this case, a reasonable possibility for exciting the nucleus. In such a scenario the previous two disadvantages are not relevant, as they are connected with nuclear recoil experiments. This possibility is indeed good news, because measuring the de-excitation γ rays is a much simpler task than the detection of the recoiling nuclei.

The above conclusions depend only on the velocity distribution and nuclear structure and are independent of the specific nature of the WIMP.

It must be emphasized that it is not easy to construct extensions of the Standard Model that include a dark energy field coupled to dark matter. The main obstruction is related to the necessity to keep the mass of the field of the order of the present Hubble scale after radiative corrections. A large coupling to the dark matter field induces significant loop corrections to the potential of the dark energy field, resulting in a large mass [42]. On the other hand, it is reasonable to expect that the resolution of the coincidence problem will require a coupling that is not much smaller than the gravitational one. For this reason our analysis will be essentially phenomenological. We will assume that the dark energy field has a mass of the order of the Hubble scale and a coupling to the dark matter of gravitational strength. Explicit models that realize these assumptions are given in refs. [43, 44].

INTERACTION BETWEEN DARK MATTER AND DARK ENERGY

We consider an interaction between the scalar field and the dark matter particles that can be modeled through a field-dependent particle mass. The action takes the form

$$\mathcal{S} = \int d^4x \sqrt{-g} \left(M^2 R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - U(\phi) \right) - \sum_i \int m(\phi(x_i)) d\tau_i, \quad (2)$$

where $d\tau_i = \sqrt{-g_{\mu\nu}(x_i) dx_i^\mu dx_i^\nu}$ and the second integral is taken over particle trajectories. Variation of the action with respect to ϕ results in the equation of motion

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \phi) = \frac{dU}{d\phi} - \frac{d \ln m(\phi(x))}{d\phi} T^\mu_\mu, \quad (3)$$

where the energy-momentum tensor associated with the gas of particles is

$$T^{\mu\nu} = \frac{1}{\sqrt{-g}} \sum_i \int d\tau_i m(\phi(x_i)) \frac{dx_i^\mu}{d\tau_i} \frac{dx_i^\nu}{d\tau_i} \delta^{(4)}(x - x_i). \quad (4)$$

We are interested in static spherically symmetric configurations, with the scalar field varying slowly with the radial distance r . Our treatment is relevant up to a distance $r_1 \sim 100 \text{ kpc}$ beyond

which the dark matter becomes very dilute. For $r \gtrsim r_1$ we expect that ϕ quickly becomes constant with a value close to $\phi(r_1) \equiv \phi_1$. This is the value that drives the present cosmological expansion. Here we assume that the cosmological evolution of ϕ_1 is negligible for the time scales of interest, so that the asymptotic configuration is static to a good approximation.

We approximate: $m(\phi) \simeq m(\phi_0) + [dm(\phi_0)/d\phi] \delta\phi \equiv m_0 + m'_0 \delta\phi$, with ϕ_0 the value of the field at the center of the galaxy ($r = 0$). We work within the leading order in $\delta\phi$ and assume that $m'/m \simeq m'_0/m_0$ for all r . Also $dU/d\phi$ can be approximated by a constant between $r = 0$ and $r = \infty$. For the scalar field to provide a resolution of the coincidence problem, the two terms in the r.h.s. of Eq. (3) must be of similar magnitude in the cosmological solution. This means that $dU/d\phi$ must be comparable to $(m'_0/m_0)\rho_\infty$. We expect ρ_∞ to be a fraction of the critical density, i.e. $\rho_\infty \sim 3 \text{ keV/cm}^3$. On the other hand, the energy density in the central region of the static solution ($r \lesssim 100 \text{ kpc}$) is that of the galaxy halo ($\sim 0.4 \text{ GeV/cm}^3$ for our neighborhood of the Milky Way). This makes $dU/d\phi$ negligible in the r.h.s. of Eq. (3) for a static configuration. The potential is expected to become important only for $r \rightarrow \infty$, where the static solution must be replaced by the cosmological one. Similar arguments indicate that we can neglect U relative to ρ . Also the scalar field must be effectively massless at the galactic scale. For these reasons we expect that the form of the potential plays a negligible role at the galactic level. Our analysis can be carried out with $U = 0$ and is model independent.

We treat the dark matter as a weakly interacting, dilute gas. We are motivated by the phenomenological success of the isothermal sphere [45] in describing the flat part of the rotation curves. We do not address the question of the density profile in the inner part of the galaxies ($r \lesssim 5 \text{ kpc}$). We approximate the energy-momentum tensor of the dark matter as $T^\mu_\nu = \text{diag}(-\rho, p, p, p)$ with $p(r) = \rho(r) \langle v_d^2 \rangle = m(\phi(r)) n(r) \langle v_d^2 \rangle$. The dispersion of the dark matter velocity is assumed to be constant and small: $\langle v_d^2 \rangle \ll 1$. The gravitational field is considered in the Newtonian approximation: $g_{00} \simeq 1 + 2\Phi$, with $\Phi = \mathcal{O}(m'_0 \delta\phi/m_0)$. In the weak field limit and for $p \ll \rho$, the conservation of the energy-momentum tensor gives

$$p' = -\rho \Phi' - \rho \frac{m'_0}{m_0} (\delta\phi)', \quad (5)$$

with the prime on p , Φ , $\delta\phi$ denoting a derivative with respect to r . Integration of this equation gives $n \simeq n_0 \exp(-\Phi/\langle v_d^2 \rangle - (m'_0/m_0)\delta\phi/\langle v_d^2 \rangle)$.

With the above assumptions we obtain the equations of motion

$$\Phi'' + \frac{2}{r} \Phi' = \frac{1}{4M^2} \rho_0 \exp(-\alpha\Phi - \tilde{\alpha}\delta\phi), \quad (6)$$

and

$$(\delta\phi)'' + \frac{2}{r} (\delta\phi)' = \frac{m'_0}{m_0} \rho_0 \exp(-\alpha\Phi - \tilde{\alpha}\delta\phi), \quad (7)$$

where $M = (16\pi G_N)^{-1/2}$ is the reduced Planck mass, $\rho_0 = m_0 n_0$ the energy density of dark matter at $r = 0$, $\alpha = 1/\langle v_d^2 \rangle$, and $\tilde{\alpha} = m'_0/(m_0 \langle v_d^2 \rangle)$. We emphasize that, even though $|\Phi| \ll 1$, the combination $\Phi/\langle v_d^2 \rangle$, that appears in the exponent in the expression for the number density n , can be large. Similarly, the expansion of the mass around the value $m_0 = m(\phi_0)$ assumes the smallness of the dimensionless parameter $|m'_0 \delta\phi/m_0|$. However, the combination $\tilde{\alpha}\delta\phi = (m'_0 \delta\phi/m_0)/\langle v_d^2 \rangle$, that appears in the exponent, can be large.

A linear combination of Eqs. (6), (7) gives

$$\frac{d^2 u}{dz^2} + \frac{2}{z} \frac{du}{dz} + \exp u, \quad (8)$$

where $u = -\alpha\Phi - \tilde{\alpha}\delta\phi$, $z = \beta r$ and $\beta^2 = (1 + \kappa^2)\alpha\rho_0/4M^2$. The parameter

$$\kappa^2 = 4M^2 (m'_0/m_0)^2 \quad (9)$$

determines the strength of the new interaction relative to gravity. The solutions that are regular for small z approach the form

$$u = \ln\left(\frac{2}{z^2}\right) + \frac{1}{\sqrt{z}} \left[d_1 \cos\left(\frac{\sqrt{7}}{2} \ln z\right) + d_2 \sin\left(\frac{\sqrt{7}}{2} \ln z\right) \right] + \dots \quad (10)$$

for large z . Another linear combination of Eqs. (6), (7) gives

$$\frac{d^2 w}{dz^2} + \frac{2}{z} \frac{dw}{dz} = 0, \quad (11)$$

with $w = -\kappa^2 \alpha \Phi + \tilde{\alpha} \delta \phi$. The solution of this equation is $w = c_0 + c_1/z$.

The velocity v of a massive baryonic object in orbit around the galaxy, at a distance r from its center, can be expressed as

$$\left(\frac{v}{v_c}\right)^2 = \frac{r\Phi'}{v_c^2} = -\frac{z}{2} \left(\frac{du}{dz} + \frac{dw}{dz} \right), \quad (12)$$

where

$$v_c^2 = \frac{2}{1 + \kappa^2} \langle v_d^2 \rangle. \quad (13)$$

The asymptotic form of $u(z)$, $w(z)$ indicates that $v \simeq v_c$ for large z . The dominant correction to the leading behavior arises from the term $\sim 1/\sqrt{z}$ in Eq. (10). The function $v(z)$ gives a higher order correction. This simple analysis indicates that the approximately flat rotation curves outside the galaxy cores are a persistent feature even if the dark matter is coupled to a scalar field through its mass. If the new interaction is universal for ordinary and dark matter, the experimental constraints impose $\kappa^2 \ll 1$. In this case, it is reasonable to expect a negligible effect in the distribution of matter in galaxy halos. However, if ϕ interacts only with dark matter, as we assume here, this bound can be relaxed significantly.

A massive particle in orbit around the galaxy, at a large distance r from its center, has a velocity given by Eq. (13). We can use this expression in order to fix $\langle v_d^2 \rangle$ for a given value of κ . The effect of the new scalar interaction is encoded in the factor κ^2 . When this is small, the velocity of an object orbiting the galaxy is of the order of the square root of the dispersion of the dark matter velocity. If κ^2 is large, the rotation velocity can become much smaller than the typical dark matter velocity.

The allowed range of κ is limited by the observable implications of the model that describes the dark sector. It is reasonable to expect that the resolution of the coincidence problem through an interaction between dark matter and dark energy will have to rely on a coupling not significantly weaker than gravity. It seems unlikely that a coupling $\kappa^2 \ll 1$ can lead to a cosmological evolution drastically different from that in the decoupled case.

The dependence of the mass of dark matter particles on an evolving scalar field during the cosmological evolution since the decoupling is reflected in the microwave background. The magnitude of the effect is strongly model dependent. In the models of ref. [37, 43] the observations result in the constraint $\kappa^2 \lesssim 0.01$. In the model of ref. [41] the scalar interaction among dark matter particles is screened by an additional relativistic dark matter species. As a result, the model is viable even for couplings $\kappa^2 \simeq 1$. A similar mechanism is employed in ref. [44]. In this model the interaction between dark matter and dark energy becomes important only during the recent evolution of the Universe. In general, an interaction that is effective for redshifts $z \lesssim 1-2$ is not strongly constrained by the observations.

Independently of the value of κ^2 , the interaction of dark matter with the scalar field associated with dark energy does not destroy the approximately flat profile of the rotation curves. Other considerations, however, could constrain the coupling κ^2 . The dispersion of the dark matter velocity is $\langle v_d^2 \rangle = (1 + \kappa^2)v_c^2/2$. For a value of v_c deduced from observations, $\langle v_d^2 \rangle$ increases with κ . For sufficiently large κ , it seems possible that v_c may exceed the escape velocity from the galaxy. It turns out, however, that this is not the case. Outside the core of the galaxy and for $r \lesssim r_1$, the

binding potential for a dark matter particle is $\Phi + (\tilde{\alpha}/\alpha)\delta\phi$. For large r , Eq. (11) implies that $v = -\kappa^2\alpha\Phi + \tilde{\alpha}\delta\phi = \text{constant}$. The binding potential becomes $(1 + \kappa^2)\Phi = (1 + \kappa^2)v_c^2 \ln(r/r_1)$, where we have omitted an overall constant. For a particle at a distance r_* from the center of the galaxy, the escape velocity becomes

$$v_{esc}^2 = 2(1 + \kappa^2)[\ln(r_1/r_*) + 1]. \quad (14)$$

The value of v_{esc} is larger than the standard one [6] by a factor $(1 + \kappa^2)$, so that $\langle v_d^2 \rangle$ remains substantially smaller than v_{esc}^2 for $r_* \ll r_1$. A particle that does not interact with the scalar field is bound only by the potential Φ . However, the scale of its velocity is set by v_c , so that again it cannot escape.

THE VELOCITY DISTRIBUTION OF DARK MATTER

In the previous section we saw that in isothermal models the dark matter velocity distribution with respect to the galactic center is M-B:

$$f(v) = \frac{1}{\pi\sqrt{\pi}} \frac{1}{v_m^3} \exp\left(-\frac{v^2}{v_m^2}\right), \quad (15)$$

where $v_m^2 = 2v_d^2 = (1 + \kappa^2)v_c^2$, with v_c the observed rotation velocity of a baryonic object in orbit around the galaxy. This means that the dispersion of the dark matter velocity is proportional to $1 + \kappa^2$, where κ is the coupling between dark matter and dark energy, given by Eq. (9). We have also assumed that the ordinary baryonic matter does not couple to the dark energy field and is not affected by its presence. We impose an upper bound v_b on the dark matter velocity, equal to the escape velocity given by Eq. (14). We express it as

$$v_b = n v_{esc,0}, \quad (16)$$

where $v_{esc,0}$ is the escape velocity for $\kappa = 0$ and $n^2 = 1 + \kappa^2$.

In the local frame this velocity distribution takes the form

$$f(\mathbf{v}) = \frac{1}{\pi\sqrt{\pi}} \frac{1}{v_m^3} \exp\left(-\frac{(\mathbf{v} + \mathbf{v}_E)^2}{v_m^2}\right), \quad (17)$$

where

$$\mathbf{v}_E = \mathbf{v}_0 + v_1(\sin\alpha \hat{\mathbf{x}} - \cos\alpha \cos\gamma \hat{\mathbf{y}} + \cos\alpha \sin\gamma \hat{\mathbf{z}}). \quad (18)$$

We have chosen a coordinate system in which the polar z-axis is along the direction of motion of the Sun, the x-axis is radially out of the galaxy and $\hat{\mathbf{y}} = \hat{\mathbf{z}} \times \hat{\mathbf{x}}$. The velocity of the Earth is \mathbf{v}_E . The velocity of the Sun around the center of the galaxy is $v_0 \hat{\mathbf{z}}$ (with $v_0 \simeq 220$ km/s). The magnitude of the velocity of the Earth relative to the Sun is $v_1 \simeq 30$ km/s. The quantity $\gamma \simeq \pi/6$ describes the orientation of the ecliptic with respect to the galactic plane. (The angle between the normals to the two planes is $\pi/2 - \gamma \simeq \pi/3$.) The parameter α denotes the phase of the Earth. ($\alpha = 0$ on June 2nd.)

In the standard scenario, in which there is no interaction between dark matter and dark energy, we have $v_m = v_0$ and $v_b = y_{esc}v_0$ with $y_{esc} \simeq 2.84$. In the scenario we are considering both parameters are scaled up by the same factor, i.e. $v_m = n v_0$ and $v_b = n y_{esc} v_0$, with $n = \sqrt{1 + k^2}$. The standard M-B distribution is a special case of our model with $n = 1$. In the present work we treat n as a free parameter, which we do not expect to be much larger than unity. We will consider values as large as $n = 2$ (that corresponds to $\kappa = \sqrt{3}$) and study the implications for direct dark matter detection. The distribution function can be written as:

$$f(y, \xi, \phi, \delta, n) = \frac{1}{\pi\sqrt{\pi}} \frac{1}{n^3} e^{\frac{-y^2 - 2((2\delta)^2 + y\sqrt{1-\xi^2} \cos(\phi) \sin(\alpha) 2\delta + \cos(\alpha)((y\xi+2) \sin(\gamma) - y\sqrt{1-\xi^2} \cos(\gamma) \sin(\phi)) 2\delta + y\xi + 1)}{n^2}}, \quad (19)$$

where ϕ is the azimuthal angle, ξ the cosine of the angle between \mathbf{v} and \mathbf{v}_0 , $y = v/v_0$, $\delta = \sin \gamma v_1/v_0 \simeq (1/2)(30/220) = 0.068$. The integral over ϕ can be done analytically to yield:

$$\tilde{f}(y, \xi, \delta, n) = \frac{2}{\sqrt{\pi}} \frac{1}{n^3} e^{\frac{-y^2 - 2((2\delta)^2 + \cos \alpha (y\xi + 2)\delta + y\xi + 1)}{n^2}} I_0 \left(2\delta y \sqrt{(1 - \xi^2)(1 - \cos^2 \alpha \sin^2 \gamma)} \right), \quad (20)$$

where $I_0(x)$ is the well known modified Bessel function. The various variables are constrained by:

$$\sqrt{y^2 + 2((2\delta)^2 + \cos \alpha (y\xi + 2)\delta + y\xi + 1)} \leq ny_{esc}. \quad (21)$$

From the kinematics of the WIMP-nucleus collision we find that the momentum transfer to the nucleus is given by

$$q = 2\mu_r v \cos \theta, \quad (22)$$

where θ is the angle between the WIMP velocity and the momentum of the outgoing nucleus, and μ_r the reduced mass of the system. Instead of the angle θ one can introduce the energy Q transferred to the nucleus, $Q = q^2/(2Am_p)$ (Am_p is the nuclear mass). Thus

$$2 \sin \theta \cos \theta d\theta = -\frac{Am_p}{2(\mu_r v)^2} dQ.$$

Furthermore, for a given energy transfer the velocity v is constrained to be

$$v \geq v_{min}, \quad v_{min} = \sqrt{\frac{QAm_p}{2}} \frac{1}{\mu_r}. \quad (23)$$

We will find it convenient to introduce, instead of the energy transfer, the dimensionless quantity u

$$u = \frac{1}{2}(qb)^2 \equiv \frac{Q}{Q_0}, \quad Q_0 = \frac{1}{Am_p b^2} = 4.1 \times 10^4 A^{-4/3} \text{ keV}, \quad (24)$$

where b is the nuclear (harmonic oscillator) size parameter.

It is clear that for a given energy transfer the velocity is restricted from below. We have already mentioned that the velocity is bounded from above by the escape velocity. We thus get

$$a\sqrt{u} \leq y \leq ny_{esc}, \quad a = \left[\sqrt{2}\mu_r b v_0 \right]^{-1}, \quad (25)$$

$$2 \sin \theta \cos \theta d\theta = -\frac{a^2}{y^2} dy. \quad (26)$$

THE DIRECT DETECTION EVENT RATE

The event rate for the coherent WIMP-nucleus elastic scattering is given by [23, 46, 47, 48]:

$$R = \frac{\rho(0)}{m_{\chi^0}} \frac{m}{m_p} \sqrt{\langle v^2 \rangle} f_{coh}(A, \mu_r(A)) \sigma_{p, \chi^0}^S \quad (27)$$

with

$$f_{coh}(A, \mu_r(A)) = \frac{100 \text{ GeV}}{m_{\chi^0}} \left[\frac{\mu_r(A)}{\mu_r(p)} \right]^2 A t_{coh} (1 + h_{coh} \cos \alpha) \quad (28)$$

In the above expression σ_{p, χ^0}^S is the WIMP-nucleon scalar cross section, $\rho(0)$ the WIMP density in our vicinity, m_{χ^0} the WIMP mass, m the target mass, A the number of nucleons in the nucleus

and $\langle v^2 \rangle = 3v_0^2/2$ the average value of the square of the WIMP velocity for $n = 1$. The number of events in time t is:

$$R \simeq 1.60 \cdot 10^{-3} \frac{t}{1\text{y}} \frac{\rho(0)}{0.3\text{GeVcm}^{-3}} \frac{m}{1\text{Kg}} \frac{\sqrt{\langle v^2 \rangle}}{280\text{km s}^{-1}} \frac{\sigma_{p,\chi^0}^S}{10^{-6}\text{pb}} f_{coh}(A, \mu_r(A)) \quad (29)$$

The quantity of interest to us is $r = t_{coh}(1 + h_{coh} \cos \alpha)$, which contains all the information regarding the WIMP velocity distribution and the structure of the nucleus. It also depends on the reduced mass of the system.

The event rate is proportional to the WIMP flux, i.e. proportional to the WIMP velocity. In Eq. (29) we have chosen to normalize the event rate using the velocity dispersion for $n = 1$, i.e. $\sqrt{\langle v^2 \rangle} = \sqrt{3/2}v_0$. As a result a compensating factor of $v/\sqrt{\langle v^2 \rangle}$ is included in r . It is not difficult to show [23, 46, 47, 48] that

$$\frac{dr}{du} = F^2(u) \int_{a\sqrt{u}}^{y_{esc}} \sqrt{\frac{2}{3}} y \frac{a^2}{y^2} y^2 dy \int_{-1}^{\xi_0(y,\alpha)} \tilde{f}(y, \xi, \delta, n) d\xi, \quad (30)$$

where $F(u)$ is the nuclear form factor. In the integrand we have displayed explicitly all the factors of y in order to keep track of their origin. The first one comes from the flux, the second from the transformation (26) and the last is the usual phase-space factor. The quantity $\xi_0(y, \alpha)$ enters because in some region of the velocity space the upper value of ξ is restricted so that the condition (21) is satisfied. The above expression can be cast in the form:

$$\frac{dr}{du} = \sqrt{\frac{2}{3}} a^2 F^2(u) \Psi(a\sqrt{u}, \alpha). \quad (31)$$

We have seen that the parameter a depends on the nucleus, v_0 and the WIMP mass.

In spite of the complications arising from the condition (21), by taking the leading order of the modified Bessel function in Eq. (20), we were able to get an analytic expression for $\Psi(x, \alpha)$ as follows:

$$\Psi(x, \alpha) = (1 - \Theta(x - y_{esc} + 1 + \delta \cos \alpha)) \Psi_{<}(x, \alpha) + \Theta(x - y_{esc} + 1 + \delta \cos \alpha) \Psi_{>}(x, \alpha) \quad (32)$$

$$\begin{aligned} \Psi_{<}(x, \alpha) = & \frac{-\text{erf}\left(\frac{y_{esc}}{n}\right) + \text{erf}\left(\frac{-x + \delta \cos \alpha + 1}{n}\right) + \text{erf}\left(\frac{x + \delta \cos \alpha + 1}{n}\right) - \text{erf}\left(\frac{2(\delta \cos \alpha + 1) - y_{esc}}{n}\right)}{2(\delta \cos \alpha + 1)} \\ & + \text{erf}\left(\frac{2(\delta \cos \alpha + 1) - y_{esc}}{n}\right) - \frac{\text{erf}\left(\frac{-y_{esc} + \delta \cos \alpha + 1}{n}\right)}{2(\delta \cos \alpha + 1)} - \frac{e^{-\frac{y_{esc}^2}{n^2}}}{n\sqrt{\pi}} \end{aligned} \quad (33)$$

$$\Psi_{>}(x, \alpha) = \frac{\frac{2e^{-\frac{y_{esc}^2}{n^2}}(x - y_{esc})}{n\sqrt{\pi}} - \text{erf}\left(\frac{-y_{esc} + \delta \cos \alpha + 1}{n}\right) + \text{erf}\left(\frac{-x + \delta \cos \alpha + 1}{n}\right)}{2(\delta \cos \alpha + 1)}, \quad (34)$$

where x is a short hand notation for $a\sqrt{u}$ and $\Theta(x)$ is the Heavyside step function.

By performing a Fourier analysis of the function $\Psi(x, \alpha)$, which is a periodic function of α , and keeping the dominant terms we find:

$$\frac{dr}{du} = \sqrt{\frac{2}{3}} a^2 F^2(u) [\Psi_0(a\sqrt{u}) + H(a\sqrt{u}) \cos \alpha + H_2(a\sqrt{u}) \cos 2\alpha]. \quad (35)$$

Sometimes we will consider separately each term in the above expression by writing:

$$\frac{dr}{du} = \frac{dt}{du} + \frac{dh}{du} \cos \alpha + \frac{dh_2}{du} \cos 2\alpha. \quad (36)$$

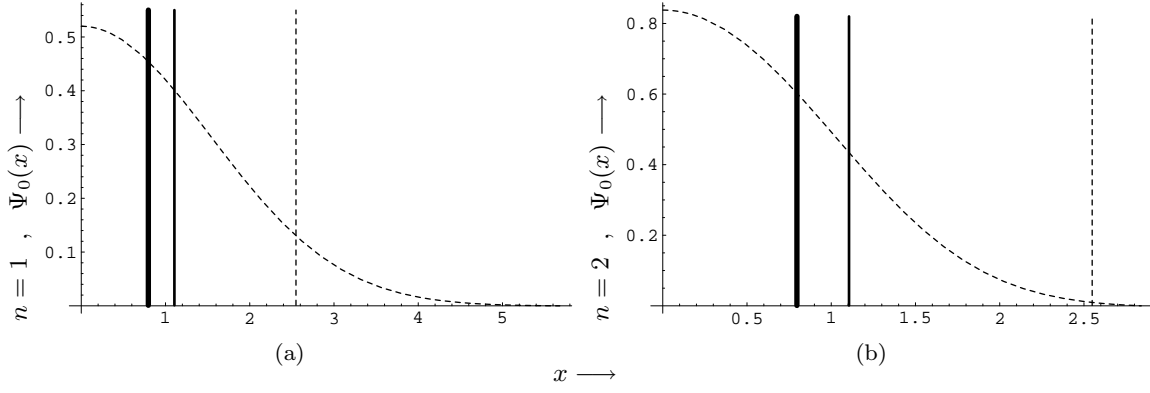


Figure 1: The function $\Psi_0(x)$ as defined in the text. From left to right $n = 1$ and 2 . The area under the curve is roughly independent of n . Due to the nuclear form factor, not all the range of u is exploitable in direct WIMP detection. For ^{127}I there is effectively a cut off value indicated by a dotted line, a fine line and a thick line for a WIMP mass of 30, 100 and 200 GeV respectively. The exploitable area under the curve decreases as n increases.

Before proceeding further by considering a special target, it is instructive to concentrate on the dependence of $\Psi_0(x)$ and the modulation $H(x)$ on the parameter n of the M-B distribution. For this purpose we exhibit the function $\Psi_0(x)$ in Fig. 1, the function $H(x)$ in Fig. 2 and $H_2(x)$ in Fig. 3. From Fig. 1 it is apparent that the high energy transfers are cut off because of the nuclear form factor at values lower than the limit imposed by the upper bound on the WIMP velocity, which increases with n . In the case of $\Psi_0(x)$, one clearly sees that the peak value decreases with n . Even though the area under the curve remains roughly independent of n , the portion available to direct detection decreases because of the nuclear form factor. In the case of $H(x)$ one notices a change in

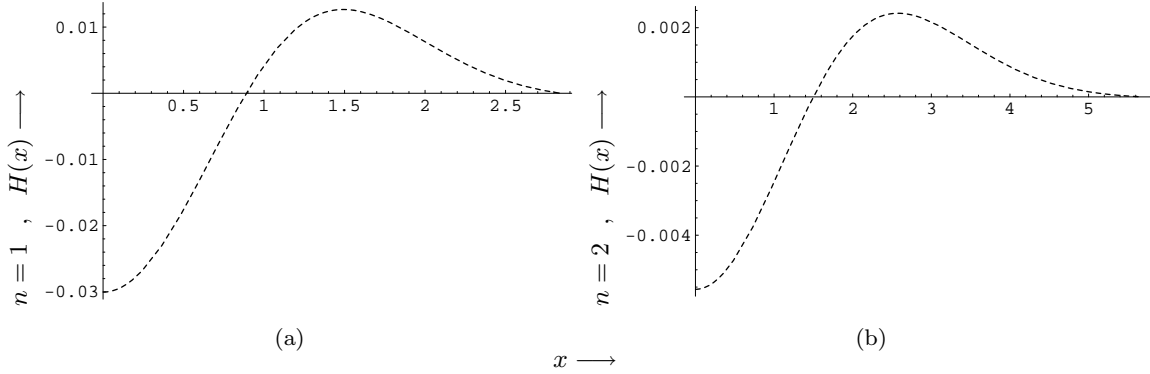


Figure 2: The function $H(x)$ giving the effect of the velocity distribution on the modulated differential rate. From left to right $n = 1$ and 2 . Note the change in sign and the fact that the amplitude decreases with increasing n .

sign. The low Q section tends to cancel the high Q part, when one integrates over Q to get the total event rate. Furthermore we see that the modulation amplitude is decreasing with n . By comparing $H(x)$ and $H_2(x)$, one can see that the effect of the higher Fourier components in α is negligible.

APPLICATIONS

As we have already mentioned, the absolute rate depends critically on the specific nature of the WIMP, e.g. on the SUSY parameters in the case of the neutralino. It also depends on the structure

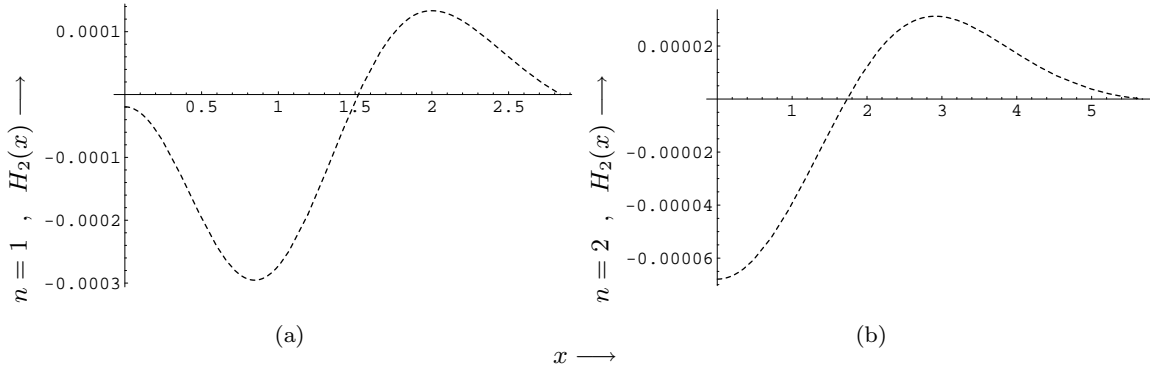


Figure 3: The function $H_2(x)$ giving a higher order effect on the modulated differential rate. From left to right $n = 1$ and 2 . Note again the change in sign and the fact that the amplitude decreases with increasing n .

of the nucleon. In the present work we will not be concerned with those very important aspects (see e.g. Refs [23, 46, 47, 48] on how one deals with such issues). The event rate is proportional to the WIMP density in our vicinity, which is not modified by including the coupling between dark matter and dark energy as in our model. In any case, we will focus here on the aspects affected by the WIMP velocity distribution.

The differential rate discussed in the previous section depends on the nucleus via its form factor and its mass. It also depends on the WIMP mass through the reduced mass μ_r entering the parameter a . For our numerical study we will focus on ^{127}I , which is one of the most popular targets employed. The nuclear form factor we use was obtained in the shell model description of the target and is shown in Fig. 4.

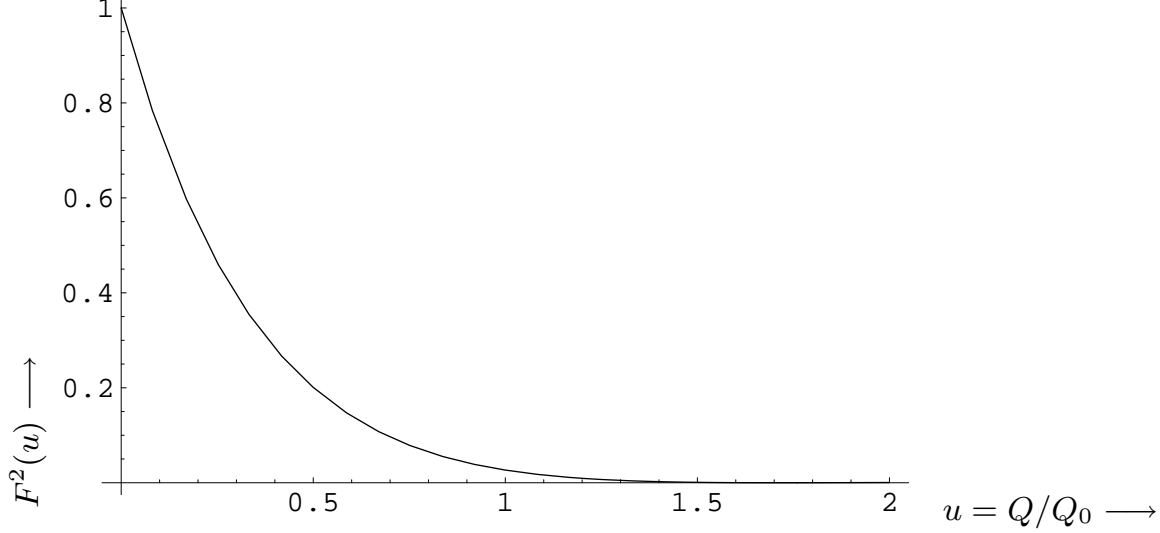


Figure 4: The form factor $F^2(u)$ employed in our calculation. u is the energy transfer to the nucleus in units of Q_0 , i.e. $u = Q/Q_0$, with $Q_0 = 64$ keV.

The part of the differential rate associated with Ψ_0 , indicated by dt_{coh}/du , is shown in Fig. 5 for two WIMP masses $m_\chi = 30$ and 100 GeV, and $n = 1, 2$. The explicit results of this figure confirm those derived by inspection of Fig. 1.

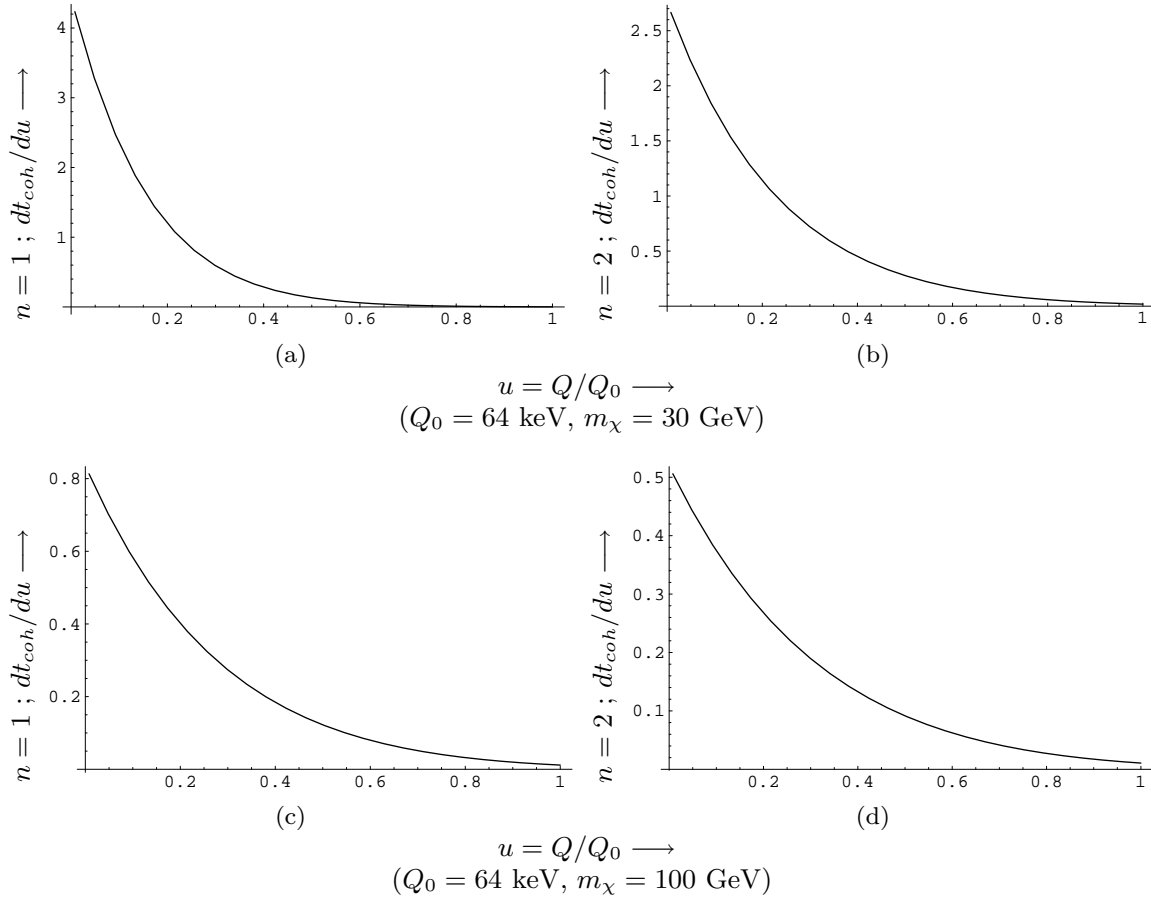


Figure 5: The quantity dt_{coh}/du for $n = 1$ on the left and $n = 2$ on the right. From top to bottom $m_\chi = 30$ and $m_\chi = 100$ GeV.

The total (time averaged) rate is given by:

$$t_{coh} = \int_{u_{min}}^{u_{max}} \frac{dt_{coh}}{du} du, \quad (37)$$

where u_{min} is determined by the detector threshold and $u_{max} = (ny_{esc})^2/a^2$ by the maximum WIMP velocity. By including both $\Psi_0(a\sqrt{u})$ and $H(a\sqrt{u})$ we can cast the rate in the form:

$$\begin{aligned} r_{coh} &= t_{coh} (1 + h_{coh} \cos \alpha) \\ h_{coh} &= \frac{1}{t_{coh}} \int_{u_{min}}^{u_{max}} \frac{dh_{coh}}{du} du. \end{aligned} \quad (38)$$

Integrating over the energy transfer, assuming either no detector cut off ($u_{min} = 0$) or a cut off of $Q_{th} = 10$ keV, we obtain the results shown in Fig. 6. One can see from Fig. 6 that, except for the case of light WIMPs, the total rate is decreasing with increasing n . The reason is that, as we have seen in the previous section, even though the total area under the curves of Fig. 1 is independent of n , the nuclear form factor dumps out the high u components. Similarly, the area under the curves of Fig. 5 decreases with n . As expected, for a given n the rate decreases as the energy cut off increases. From Fig. 6 we see that the modulation amplitude h rapidly decreases as n increases. This is expected in view of Fig. . For a given n the modulation amplitude increases as the energy cut off increases. The reason is that h essentially is the ratio of the modulated amplitude

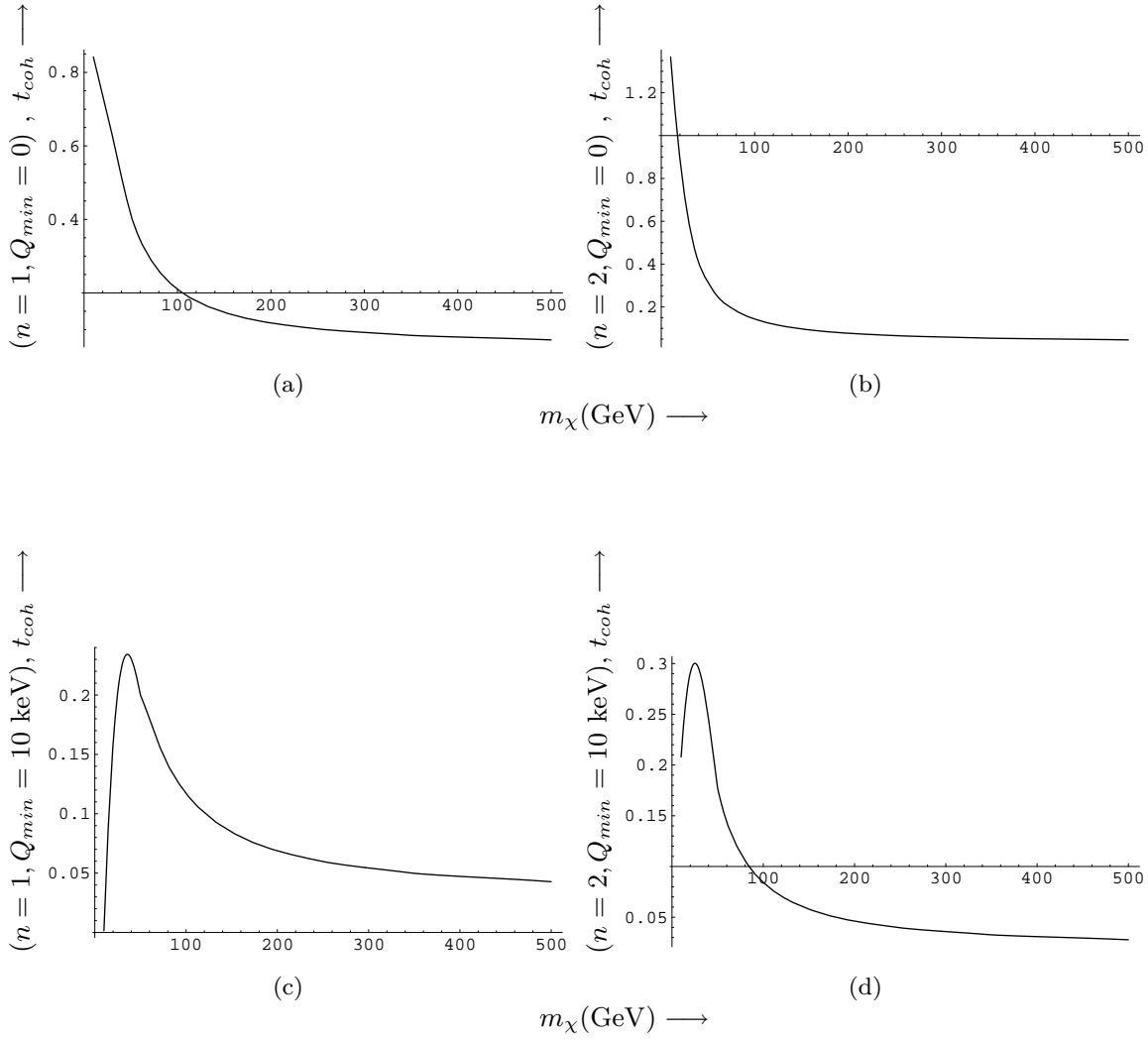


Figure 6: The quantity t_{coh} for $Q_{min} = 0$ at the top and $Q_{min} = 10$ keV at the bottom. From left to right $n = 1$ and 2.

divided by the unmodulated amplitude. Both decrease with increasing Q_{min} , but the denominator is decreasing much more rapidly. In other words the increase in the modulation occurs at the expense of the number of counts.

CONCLUSIONS

According to our present understanding of the evolution of the Universe, the main contribution to its energy content comes from two sectors, the dark energy and dark matter, that have not been directly observed. Within the majority of the models that have been proposed for the description of these sectors, there is no coupling between them other than the gravitational one. In general, this very strong assumption is not supported by some reasoning based on fundamental properties of the model, such as symmetries. On the other hand, a coupling between the two sectors may even be desirable, as it may provide an explanation of the coincidence problem, i.e. the comparable contributions of the two sectors to the energy density of the Universe today.

The main motivation for this paper has been the wish to explore the direct observational con-

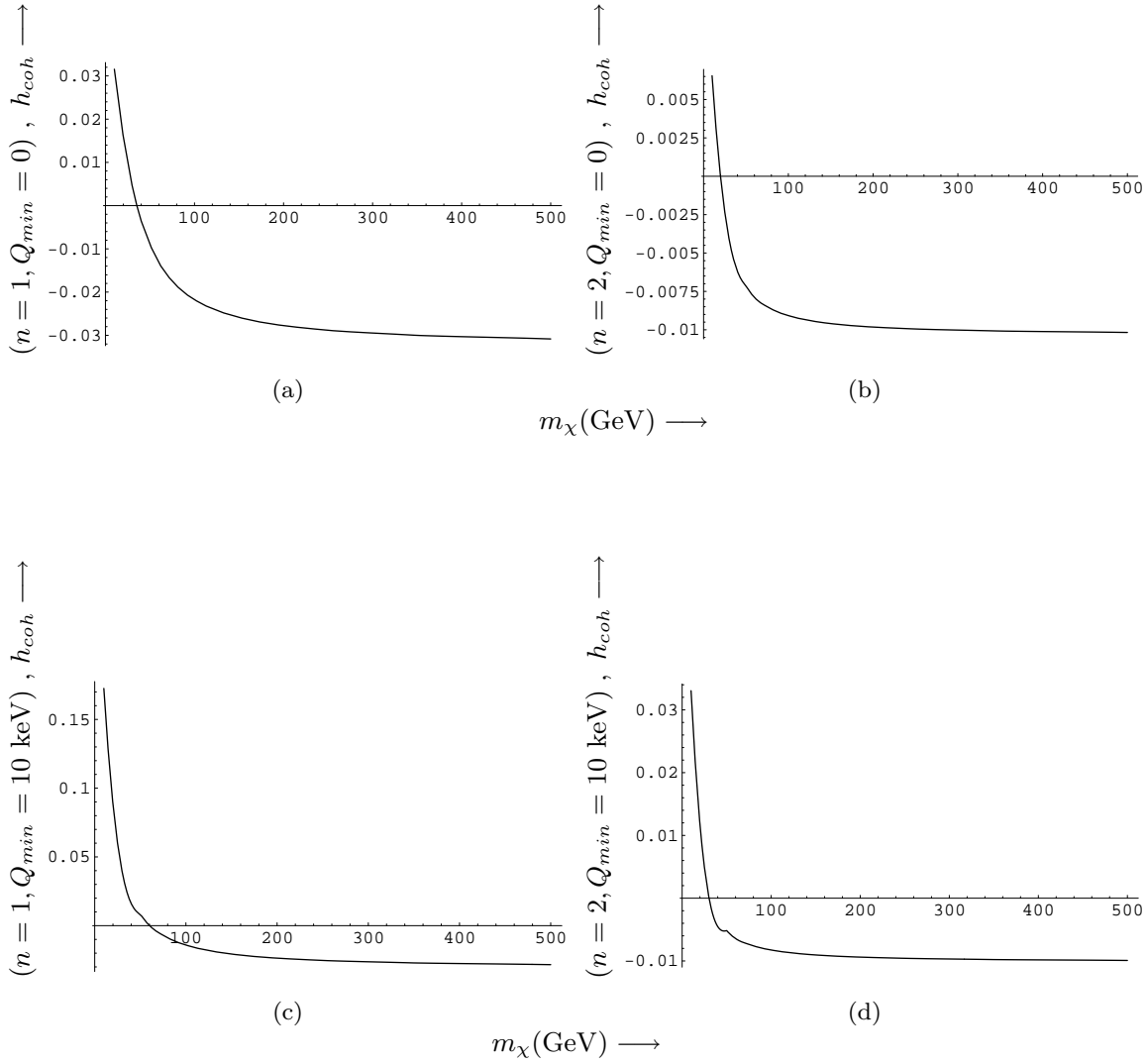


Figure 7: The quantity h_{coh} for $Q_{min} = 0$ at the top and $Q_{min} = 10$ keV at the bottom. From left to right $n = 1$ and 2.

sequences of such a coupling. We modeled the interaction between dark matter and dark energy by assuming that the mass of the dark matter particles depends on the scalar field whose potential provides the dark energy. The fact that the dark energy field, if it plays a dynamical role in the cosmological evolution today, must be effectively massless at length scales below the horizon means that its presence results in a long-range attractive force in the dark matter sector. This can have significant implications for the mechanisms of structure formation. The effect that has been of interest to us is the modification of the standard isothermal Maxwell-Boltzmann distribution of dark matter in the galaxy halos [32, 33]. The main modification is that the characteristic dark matter velocity can be increased significantly in the presence of the additional force. As the velocity affects directly the detection rates of the various experiments that search for dark matter, a detailed calculation of these rates, taking into account the new interaction, is important.

The modification of the velocity distribution has consequences for direct WIMP detection. Regarding the (time averaged) event rates, our results depend on the WIMP mass. For light WIMPs we find an increase of the rates by about 50%, independently of the detector energy cutoff, if the new force is stronger than the gravitational force by a factor $\kappa = \sqrt{3}$. For larger masses, however,

the new force leads to a substantial decrease in the rates. The reason for this is that large energy transfers are inhibited by the nuclear form factor. The consequences of the new interaction are more pronounced in the case of the modulation amplitude. For light WIMP masses the modulation is decreased by an order of magnitude for an ideal detector (zero energy threshold) for $\kappa = \sqrt{3}$ relative to $\kappa = 0$. For heavy WIMPs the decrease is about a factor of 4. The effect persists, but is somewhat less pronounced, in the case of a detector with a finite energy threshold, e.g. about 10 keV.

We should emphasize that the average WIMP energy, in addition to its linear increase with the WIMP mass, also increases proportionally to $1 + \kappa^2$ (see Eq. (1)). The same holds for the maximum WIMP energy, which depends on the escape velocity and scales with the same factor (see Eq. (16)). This provides the opportunity of planning novel experiments other than those involving nuclear recoil, e.g. experiments detecting transitions to excited nuclear states in the MeV region. The relevant rate may be enhanced significantly due to the tail of the velocity distribution.

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